After taking a programming class, FJ started writing and selling apps. Revenues during the first month were small. **SCORE: ____ / 25 PTS**
In the third month, revenues were $\$144$, and in the sixth month, revenues were $\$486$. If FJ's monthly revenues formed a geometric sequence, find the total revenues for the first one and a half years.

$a_3 = a_1 r^2 = 144$

$a_6 = a_1 r^5 = 486$

$\frac{a_1 r^5}{a_1 r^2} = \frac{486}{144}$

$r^3 = \frac{27}{8}$

$r = \frac{3}{2}$

$a_1 \left(\frac{3}{2}\right)^2 = 144$

$a_1 = 64$

$S_{18} = \frac{64\left(\left(\frac{3}{2}\right)^{18} - 1\right)}{\frac{3}{2} - 1}$

$= \$189,042.16$

A circle has $(-1, -6)$ and $(7, -2)$ as endpoints of a diameter. **SCORE: ____ / 25 PTS**

[a] Find parametric equations for the circle.

CENTER $= \left(\frac{-1+7}{2}, \frac{-6+-2}{2}\right) = (3, -4)$

RADIUS $= \frac{1}{2}\sqrt{(7--1)^2 + (-2--6)^2} = \frac{1}{2}\sqrt{64+16} = 2\sqrt{5}$

$x = 3 + 2\sqrt{5}\cos t$

$y = -4 + 2\sqrt{5}\sin t$

[b] A particle moves counterclockwise around the circle, starting at the rightmost point of the circle. If the particle makes one complete circle every 10 seconds, find parametric equations for the particle's position.

PERIOD $= 10$

$x = 3 + 2\sqrt{5}\cos\frac{2\pi}{10}t = 3 + 2\sqrt{5}\cos\frac{\pi t}{5}$

$y = -4 + 2\sqrt{5}\sin\frac{2\pi}{10}t = -4 + 2\sqrt{5}\sin\frac{\pi t}{5}$

Use sigma notation to write the series $\frac{91}{17}-\frac{88}{19}+\frac{85}{21}-\cdots-\frac{10}{71}$. ← ARITHMETIC $d=-3$ ← ARITHMETIC $d=2$

SCORE: ____ / 20 PTS

Show clearly how you found the upper limit of summation (ie, not just by counting).

$$\sum_{i=1}^{28}(-1)^{i+1}\frac{91-3(i-1)}{17+2(i-1)}$$

$$=\sum_{i=1}^{28}(-1)^{i+1}\frac{94-3i}{15+2i}$$

$91-3(i-1)=10$

$-3(i-1)=-81$

$i-1=27$

$i=28$

OR

$17+2(i-1)=71$

$2(i-1)=54$

$i-1=27$

$i=28$

Consider the sequence defined recursively by $a_n=9n-2a_{n-1}$, $a_1=5$.

SCORE: ____ / 15 PTS

[a] Find the first 5 terms of the sequence, and write them as a list.

$a_2=9(2)-2a_1=18-2(5)=8$

$a_3=9(3)-2a_2=27-2(8)=11$

$a_4=9(4)-2a_3=36-2(11)=14$

$a_5=9(5)-2a_4=45-2(14)=17$

5, 8, 11, 14, 17

[b] Based on the first 5 terms, is the sequence arithmetic, geometric or neither ? Justify your conclusion **very briefly**.

ARITHMETIC $d=3$

Eliminate the parameter to find rectangular equations corresponding to the parametric equations $\begin{aligned}x&=\ln 2t\\ y&=2t^2\end{aligned}$.

SCORE: ____ / 15 PTS

Write *y* as a function of *x*.

$x=\ln 2t$

$e^x=2t$

$t=\frac{1}{2}e^x$

$y=2\left(\frac{1}{2}e^x\right)^2$

$y=2\left(\frac{1}{4}e^{2x}\right)$

$y=\frac{1}{2}e^{2x}$

Prove by mathematical induction: $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$ for all integers $n\geq 1$. **SCORE:____ / 30 PTS**

PROOF: BASIS STEP $n=1$

$\frac{1}{1\times 2}=\frac{1}{1+1}$

$\frac{1}{2}=\frac{1}{2}$ ✓

INDUCTIVE STEP

ASSUME $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{k(k+1)}=\frac{k}{k+1}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k\geq 1$

PROVE $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$

$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$

$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$

$=\frac{k(k+2)+1}{(k+1)(k+2)}$

$=\frac{k^2+2k+1}{(k+1)(k+2)}$

$=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$

SO, BY MI,

$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

FOR ALL INTEGERS $n\geq 1$

Find the coefficient of x^8y^{63} in the expansion of $(2x^2-y^3)^{25}$. **SCORE:____ / 20 PTS**

GENERAL TERM $=\binom{25}{r}(2x^2)^{25-r}(-y^3)^r$

$=\binom{25}{r}2^{25-r}(-1)^r x^{50-2r}y^{3r}$ $\quad 3r=63$, $r=21$

$\binom{25}{21}2^4(-1)^{21}=\frac{25!}{21!\,4!}(16)(-1)$

$=\frac{25\cdot 24\cdot 23\cdot 22\cdot 21!}{21!\ 24}(-16)$

$=-202{,}400$